Diffraction limit calculation induced by aperture using photoemission secondary electron distribution

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Diffraction effects arise from the wave nature of the electron, and cause a fundamental limit to the resolving power of electron microscopes, just as the wavelength of light limits the resolution of optical microscopes. The intensity distribution of a point object is spread out in the image plane due to diffraction and causes the image blurring. A common special case is the Airy disk, which is formed when monochromatic radiation is imaged and the pupil aperture is circular and uniformly illuminated. For systems using monochromatic radiation, the diffraction limit is simply estimated by using the Rayleigh criterion without taking aberrations into account. The blur due to aberrations or any other source is then added in quadrature. The Rayleigh definition of resolution is the closest separation for two point objects of equal brightness such that the image profile shows a minimum between the two objects. For the Airy case, the image-side resolution by this definition is given by

$$\delta_d = 0.61\lambda / \alpha_d \qquad (1)$$

where λ and α_d are the wavelength and aperture half angle at the image side, respectively.

In our case, eq. (1) cannot be applied directly because the electrons from the sample have a wide distribution of energies, each with its angular distribution. The diffraction patterns from the different energies add incoherently to yield the final point image. We calculated the diffraction effect based on a realistic distribution of secondary electron energies and angles.

We assume that the angular distribution of secondary electron is given by the Lambert law:

$$P_\Omega(\theta,\phi) \propto \cos(\theta) \qquad (2)$$

where θ and ϕ are the polar and azimuthal angles of emission. We assume that this distribution holds for all energies. An electron emitted from the sample winds up at the back focal plane at a position which depends on its transverse momentum, hence its wavevector. We thus need the distribution not in θ, ϕ space but in k_x,k_y space, where k_x and k_y are components of the wavevector. The mapping from angles to wavevector is given by:

$$\begin{aligned} k_x &= K\sqrt{E}\sin\theta\cos\phi \\ k_y &= K\sqrt{E}\sin\theta\sin\phi \end{aligned} \qquad (3)$$

where K is the energy-momentum conversion factor whose numerical value is 5.32nm-eV$^{-1/2}$. E is the electron energy as emitted from the surface.

Thus, the probability distribution in k-space is defined by:

$$P_\Omega(\theta,\phi)d(\cos\theta)d\phi = P_k(k_x,k_y)dk_x dk_y \qquad (4)$$

Evaluating the Jacobian, we find that the distribution P_k is:

$$P_k = \frac{P_\Omega}{\left|\frac{\partial(k_x,k_y)}{\partial(\theta,\phi)}\right|} = \frac{P_\Omega}{\begin{vmatrix} \frac{\partial k_x}{\partial\cos\theta} & \frac{\partial k_x}{\partial\phi} \\ \frac{\partial k_y}{\partial\cos\theta} & \frac{\partial k_x}{\partial\phi} \end{vmatrix}} = \frac{P_\Omega}{K^2 E\cos\theta} \propto 1/E \qquad (5)$$

Eq.(5) tells us that the Lambertian angle distribution is very convenient because for a given energy , it results in an uniform distribution of transverse momentum within a circle of radius determined by the energy. Thus, each energy contributes an Airy disk. We can get the image-side distribution for a point object by adding up the contributions for each energy incoherently, with a weighting given by the energy distribution and the fraction of the electrons which get through the aperture for the given energy. We assume the usual model distribution for secondaries, $P_E \propto E/(E+w_f)^4$. The image for one energy is given by

$$I_E(r) = k_{\max}^2 \left(\frac{J_1(\tilde{k}r)}{\tilde{k}r} \right)^2 \qquad (6)$$

where $k_{\max} = K\sqrt{E}$ and $\tilde{k} = \min(k_{\max}, k_a)$, The cutoff momentum imposed by the aperture is given by $k_a = K\sqrt{V_{column}}\alpha_{img} = K\sqrt{V_{column}}\frac{r_a}{f}$. Here, V_{column} is the microscope column voltage, r_a is the aperture radius, and f is the image side focal length. Note that the above distribution is referred to the object plane, not the image plane.

We can thus get the point-spread function (image of a point object), referred to the object plane, by averaging (6) over energies with the weighting as described above:

$$I(r) = \left\langle \frac{\tilde{k}^4}{k_{\max}^2} \left(\frac{J_1(\tilde{k}r)}{\tilde{k}r} \right)^2 \right\rangle_E \qquad (7)$$

Similarly, the transmission of the aperture is given by

$$T = \left\langle \frac{\tilde{k}^2}{k_{\max}^2} \right\rangle_E \qquad (8)$$

Using eq.(7), the intensity distribution for a diameter=24 μm aperture, which is put at the back focal plane (BFP) of our objective lens, is shown in fig.1. The calculation was done by generating 10^5-10^6 random energies distributed as per the model and simply averaging the quantities in angle brackets. Rather than computing the Rayleigh limit as a function of aperture diameter, we made the approximation of defining the resolution as the full width at half maximum, a quantity which is much easier to evaluate. It can be easily shown that the Rayleigh criterion (splitting double stars) yields a more optimistic value than the FWHM of the intensity distribution. The FWHM is thus a conservative estimate.

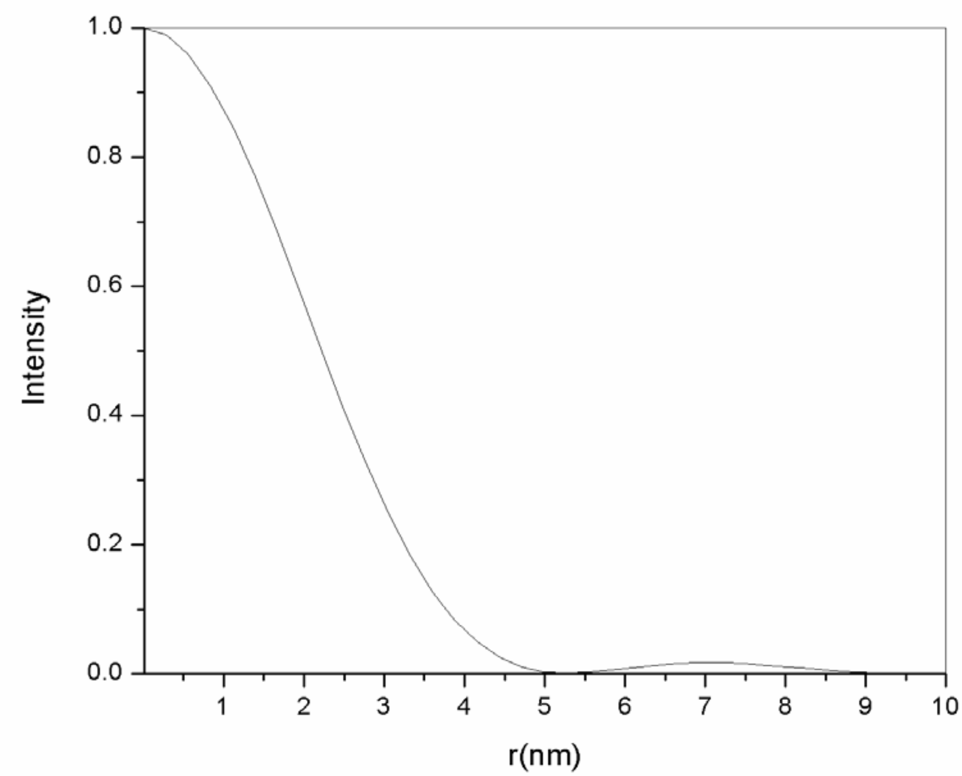

Fig.1 Intensity distribution for diameter=24 μm aperture at BFP of objective lens

Thus, the system resolution, referred to the object plane, is given by a quadrature addition:

$$r = \sqrt{r_{raytrace}^2 + d_{FWHM}^2} \qquad (9)$$

where $r_{raytrace}$ is the resolution by raytrace calculation. The resolution for PEEM3 is given in figure 2. It can be seen that 4-5 nm resolution can be reached with 1% transmission for PEEM3.

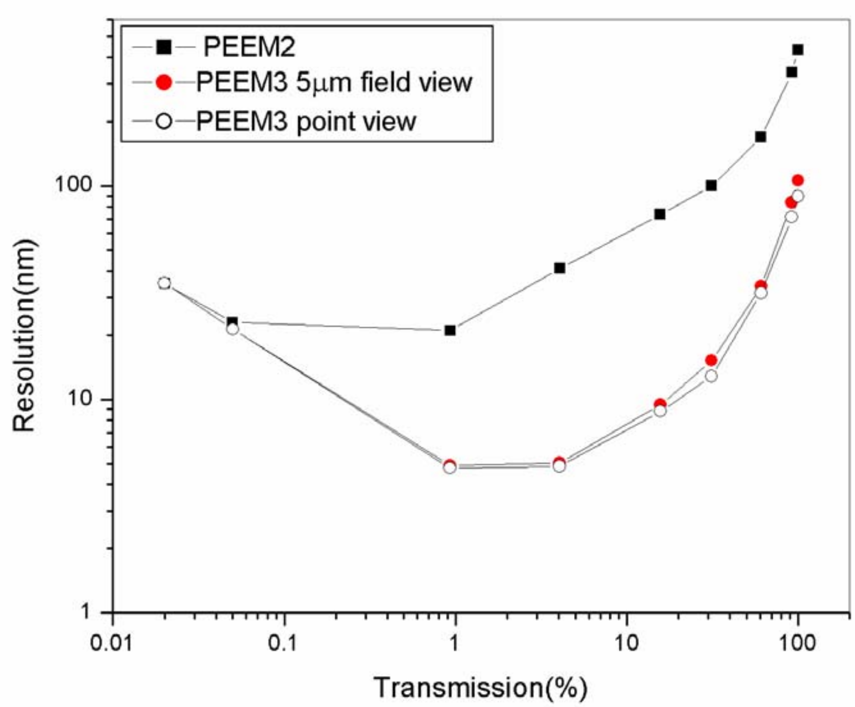

Fig.2 resolution vs transmission for m=10 front end and a diode mirror.

If the aperture is put at the back focal plane of the acceleration+objective lens, the transmission can also be derived from the following integral, which, surprisingly, can be evaluated in simple closed form

$$T = 2\pi \int_0^{\pi/2} \int_0^{E(\alpha)} \frac{E(E+W_f)^{-4} \cos\alpha \sin\alpha d\alpha dE}{\pi/(6W_f^2)}$$

$$= 1 - \frac{1}{[1+(\frac{a}{f_i^*})^2 \frac{U}{W_f}]^2} \text{......(10)}$$

Eq.(10) shows that the transmission is a function of the aperture size, image focal length, sample potential and work function. However, the diffraction-limited resolution expressed as a function of transmission depends only on the work function and not on the other quantities. This is because the transmission and resolution both depend only on the maximum transverse momentum allowed by the aperture and on the energy distribution from the sample. Of course, these conclusions only apply to the diffraction part of the blur. If one changed the aperture size and focal length in proportion, the transmission and diffraction limits wouldn't change, but the aberrations might.

The comparison of eq.(10) (in real space) and eq(8) (in momentum space) is shown in fig.3. It can be seen that the agreement is very good, which shows that these two methods are consistent with each other.

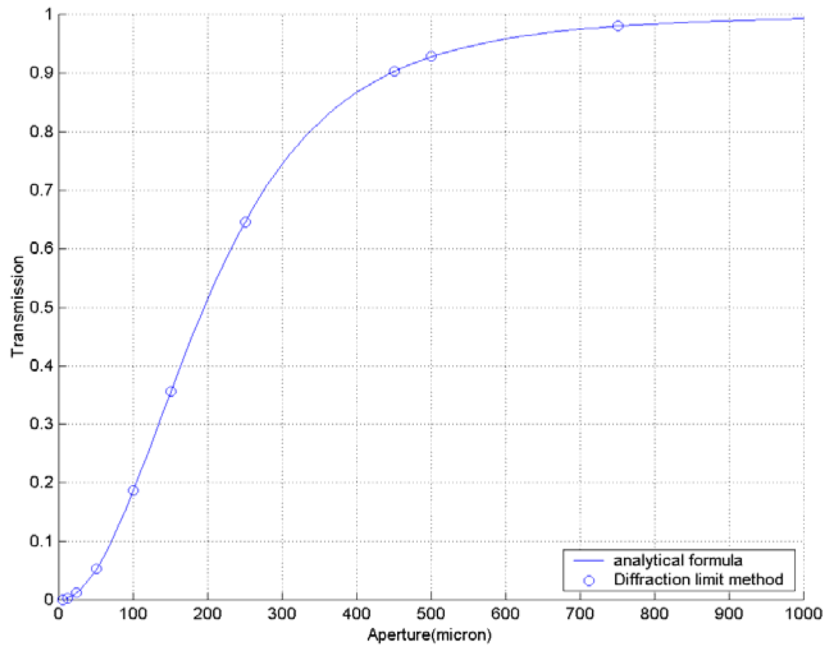

Fig.3 comparison between analytical formula and momentum space method